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Introduction to Focus Issue: Two-Dimensional Turbulence

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I. BY A. S. LANOTTE

The paper "Inertial Ranges in Two-Dimensional Turbulence" by Robert H. Kraichnan appeared in Physics of Fluids in 1967.[1] Fifty years later, with 17 invited papers from 43 different authors, this focus issue on Two-Dimensional Turbulence is both a celebration of a seminal contribution to the theoretical understanding of turbulence and a partial overview of the profound and vast legacy of the work of Kraichnan. It is a great pleasure to thank the Guest Editors, Professors Guido Boffetta, Gregory Falkovich, and Michael Shats, and let them introduce the contributions they have solicited.

II. BY G. FALKOVICH, G. BOFFETTA, AND M. SHATS

The paper "Inertial Ranges in Two-Dimensional Turbulence," for which Kraichnan received the Dirac Medal, is a seminal contribution into theory of turbulence and fluid mechanics in general. Breathtaking conceptual novelty of the paper is that it turns turbulence upside down and we still live in this topsy-turvy world. Indeed, it is quite natural to think about turbulence as an incessant process of fragmentation and mixing, which dissipates energy and wipes away details. Kraichnan has shown that two-dimensional turbulence exhibits an inverse energy cascade, transmitting energy to larger and larger scales in the process of counter-intuitive turbulent self-organization.

The idea that conservation of squared vorticity added to the energy conservation requires some upscale energy transfer in two dimensions was expressed before 1967 by Onsager, Batchelor, and Fjortof. However, even in three dimensions, there is some transfer of energy towards scales larger than the integral scale as a consequence of the redistribution of energy among all modes on the way to equilibrium. A non-trivial question is what happens in a steady far-from-equilibrium state. The idea that in such states the flow can exhibit an inverse energy cascade was first suggested by Kraichnan in this seminal paper. It is particularly admirable that this paper does not use any closure, the scourge of most of turbulence theory. Kraichnan's paper also presents the analysis of three-mode interactions, analytics on the evolution of Gaussian initial conditions, and prediction of "turbulent condensation" analogous to the quantum Bose-Einstein condensation (the last sentence of the paper).

This jubilee Focus Issue shows how seminal are the two ideas put forward in the Kraichnan paper—inverse energy cascade and condensation. Note how different papers mention and stress different aspects of Kraichnan's legacy. The collection provides a broad picture of present-day activities in fundamental turbulence studies and in applications.

Since our world is essentially three-dimensional, an important and recurring topic is where and how the notions of two-dimensional turbulence can be applied. Two complementary reviews by Xia-Francois[2] and Musacchio-Boffetta[3] discuss respectively experiments and modeling of turbulence in fluid layers. It is quite non-trivial how almost two-dimensional turbulence with an inverse cascade can appear in three-dimensional layers and co-exist with a direct energy cascade. Another interesting insight into transitions between 2D and 3D turbulence is presented in the review by Biferale, Buzzicotti, and Linkmann,[4] where a synthetic two-dimensional-three-component turbulence is considered. The subject of a split energy cascade presented by Musacchio and Boffetta[3] is to be compared with the dual constant-flux energy cascade in the review by Pouquet *et al.*[5] The latter is a broad review where dual cascades are presented in the context of atmospheric and oceanic observations, direct numerical simulations, and modeling. New data are presented for rotated stratified turbulence, and the subject is further explored in the review of Oks *et al.*[6] Here the focus is on the non-monotonic dependence of the strength of the inverse cascade on the relative role of rotation versus stratification and suppression of resonant wave triads as a mechanism for that.

The review by Frishman[7] explores the condensation phenomenon, where some unexpected and rare advances in analytical theory happened recently. At least in some cases, one can derive analytically the profile of the mean flow and form of the Reynolds stress tensor. This line of work opens interesting possibilities for further progress and may or may not be only in two dimensions. The analytical approach is essentially perturbative, based on considering the scales of fluctuations for which typical inverse turnover time if less than the mean flow gradient. Tobias and Marston[8] demonstrate the remarkable possibilities of this approach by employing direct statistical simulations of the respective perturbative equations on the mean flow and correlation functions. The common thread running through these two reviews is whether the mean flow is organized into jets or vortices, or both. The review by Bouchet *et al.*[9] takes this (perturbative or quasilinear) approach even further and uses it not only to study mean values and low-order moments but to infer the probability of large fluctuations, most importantly computing the large deviation rate-function for the Reynolds stress of a zonal jet on a rotating sphere.

Kraichnan's theory was essentially spectral and did not include coherent vortices. The article by Burgess *et al.*[10] surveys the role of coherent vortices in two-dimensional turbulence. It illustrates how vortices spontaneously formed in the decaying turbulence trap enstrophy and violate Batchelor's self-similarity assumption that energy alone determines the long-time evolution of the spectrum. As far as forced turbulence is concerned, they claim that only the incoherent part of the inverse cascade satisfies Kraichnan's scaling and the full spectrum rises more steeply.

Fittingly, the review from Japan on the Rashomon effect (by Cerbus and Chakraborti[11]) is devoted to the third-order velocity structure function which is a truly multi-faceted object in 2D flows. It expresses both energy and enstrophy fluxes and reflects the influence of both cascades, large-scale drag, dissipative anomalies, and turbulence non-stationarity. Shifting focus from spectra to fluxes is a welcome development in present-day turbulence studies, and this review shows daunting complexities of flux analysis in two dimensions.

Inverse cascade in two dimensions makes boundaries much more important than in three-dimensional turbulence. In particular, no-slip walls must significantly enhance dissipation. How vortices collide with walls and what dissipation this produces is described in the review by Clercx and van Heijst.[12]

The review by Fang and Ouellette[13] describes experimental data on the turbulence decay in fluid layers, where they identified three distinct stages of exponential decay.

Inverse cascades exist also in other turbulent systems, such as turbulent convection. The review by Mazzino[14] discusses results from numerical simulations of the inverse cascade in Rayleigh-Taylor and in homogeneous Rayleigh-Bénard two-dimensional turbulence in a unifying description based on the Bolgiano-Obukhov scaling for velocity and temperature fluctuations.

The review by Kuksin and Shirikyan[15] is devoted to recent mathematical proofs of existence, uniqueness, and mixing of two-dimensional Navier-Stokes equations and gives rigorous support to the application of a 2D model for a thin 3D fluid layer.

On the way towards applications, the review by Lindborg and Mohanan[16] presents a surprisingly rich toy model for large-scale geophysical turbulence based on the shallow water model. The output of the model is compared with the simulation of a full general circulation model. The emphasis here is on distinguishing fluxes of kinetic and potential energies which may flow in the same direction in the direct cascade or create a flux loop at large scales (also observed in turbulence in fluid layers).

The contribution by Pandit *et al.*[17] presents an extensive review on the dynamics of complex two-dimensional fluids, including conducting fluids, fluids with inertial particles and polymer additives, binary mixtures, and superfluids.

Recent experimental discoveries of the inverse energy cascade in flows not intuitively perceived as two-dimensional broaden the initial scope of Kraichnan's paper, trigger new questions, and offer new applications. Two-dimensionality of turbulence driven by waves at the liquid-gas interface suggests new approaches to the wave disorder on the water surface and novel interpretations of transport on the liquid surfaces perturbed by waves (see Xia and Francois[2]). The existence of two inertial intervals in 2D turbulence in soap film's experiments enables new approaches to study the link between spectra of the small-scale fluctuations and large-scale friction drag. This spectral link, the comparison between mean flow profiles in 3D and 2D channel flows, as well as the dynamics of vortices in soap bubbles is reviewed by Kellay.[18] These emerging applications will ensure enduring interest to Kraichnan's seminal idea.

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