CNR Consiglio Nazionale delle Ricerche
ISAC Istituto Nazionale di Scienze dell'Atmosfera e del clima

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Lagrangian Relative Dispersion: From Observations to Modeling

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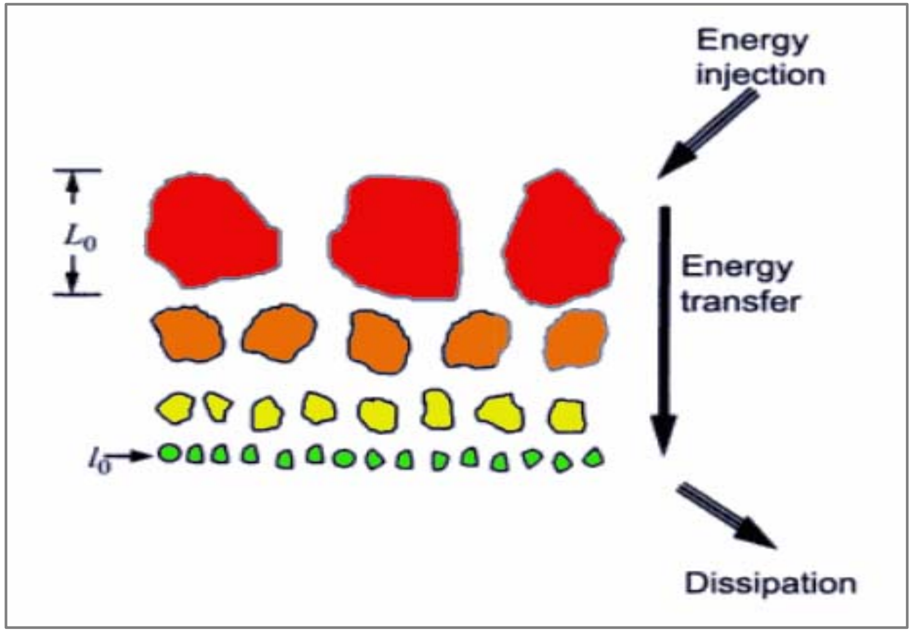
Let's make it simple..3D isotropic turbulence

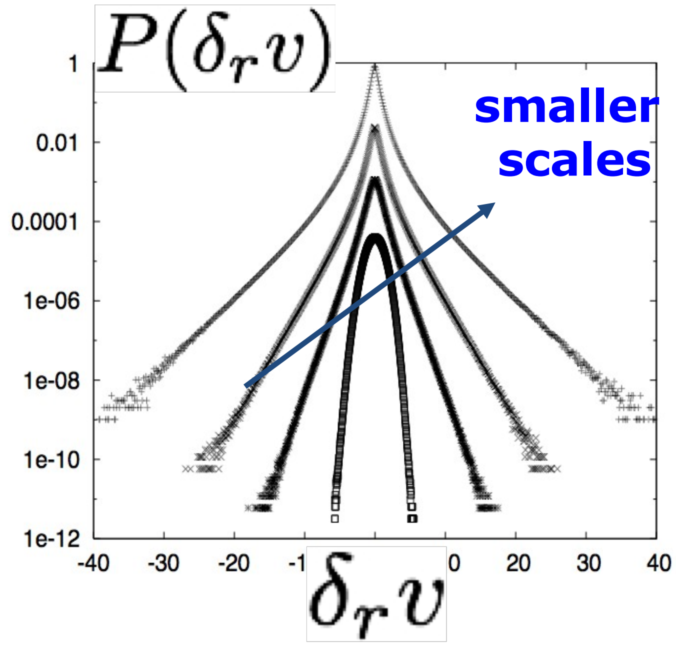
$$\partial_t \vec{\mathbf{u}} + (\vec{\mathbf{u}} \cdot \vec{\nabla})\, \vec{\mathbf{u}} = -1/\rho_0 \vec{\nabla} p + \nu \nabla^2 \vec{\mathbf{u}} + \vec{F}$$

$$\vec{\nabla} \cdot \vec{\mathbf{u}} = 0$$

$$Re = \frac{U\, L}{\nu}$$

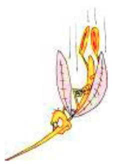
Re ~ 10^8-10^9 very high!





MULTI-SCALE PROCESS WITH STRONG BUT RARE FLUCTUATIONS
e.g. ***VELOCITY GRADIENTS***

TURBULENCE IS MUCH WORST IF YOU RIDE A PARTICLE!

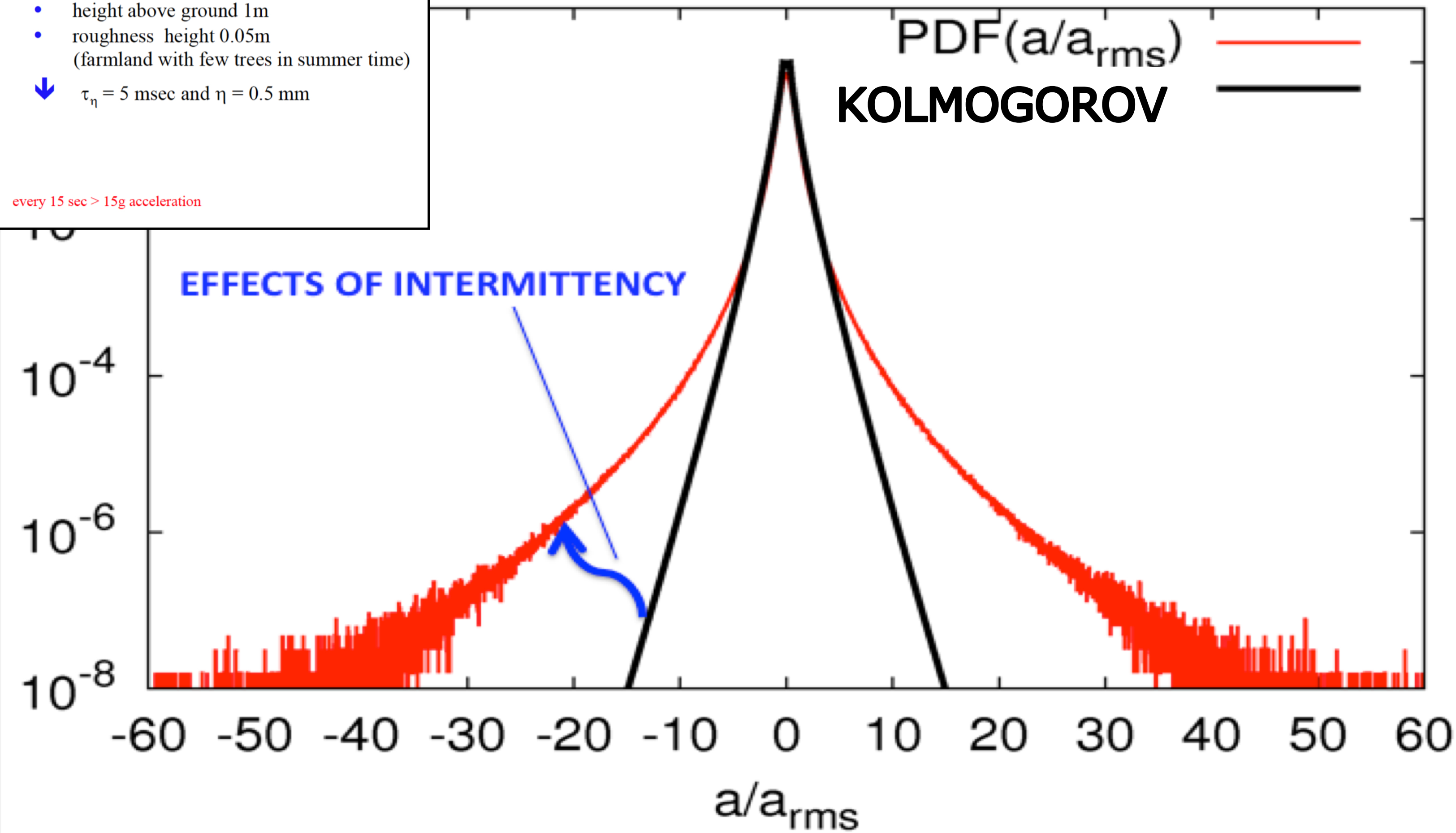


- wind speed 18km/h (5m/sec)
- height above ground 1m
- roughness height 0.05m (farmland with few trees in summer time)

τ_η = 5 msec and η = 0.5 mm

every 15 sec > 15g acceleration

TURBULENT LAGRANGIAN ACCELERATION



ACCELERATION PROBABILITY DISTRIBUTION FUNCTION (PDF) AT RE ~ 10^5 [Bi04] COMPARED WITH THE PREDICTION FROM MEAN FIELD (KOLMOGOROV THEORY)

Which are the difficulties in studying Lagrangian dispersion ?

1 need to accurately know the spatial statistics of the flow velocity along Lagrangian paths *high-resolution, high frequency*

2 need scale separation to disentangle different dispersion regimes: *exponential, ballistic, turbulent, mean shear, Taylor-like,..*

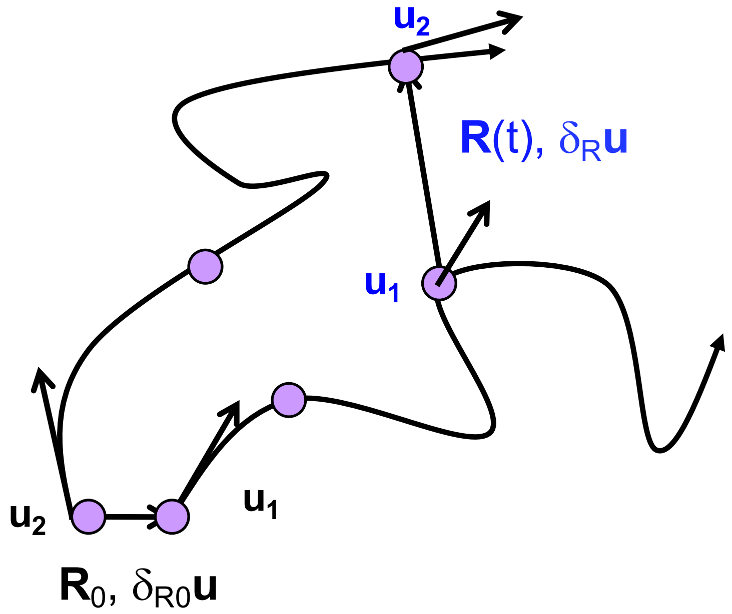
3 need to have high statistical accuracy : *long records* along *many* Lagrangian paths

4 need to limit the impact of *inhomogeneities* (walls, borders), *unsteadiness, anisotropies, stratification*

DIFFICULT TO FULLFILL ALL THESE!

So observations have to be carefully examined

How pairs of particles separate in flows ?



$$\mathbf{R}(t) = \mathbf{X_1}(t) - \mathbf{X_2}(t)$$

$$\frac{d\mathbf{R}}{dt} = \mathbf{u}_1(\mathbf{X}_1, t) - \mathbf{u}_2(\mathbf{X}_2, t)$$

INGREDIENTS

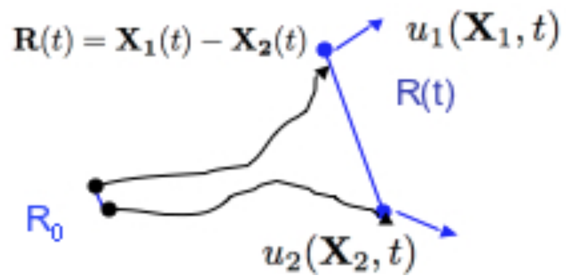
- Pair separation $\mathbf{R}_0$ at time t=0
- Pair relative velocity at t=0: $\delta_R\mathbf{u}(t=0) = \mathbf{u}_1(\mathbf{X}_1,t=0) - \mathbf{u}_2(\mathbf{X}_2,t=0)$
- Flow velocity statistics at t>0 : $\delta_R\mathbf{u}(t) = \mathbf{u}_1(\mathbf{X}_1,t) - \mathbf{u}_2(\mathbf{X}_2,t)$
- If other forces act on the particles, also need : $\delta_R\mathbf{a}(t) = \mathbf{a}_1(\mathbf{X}_1, \mathbf{u}_1, t) - \mathbf{a}_2(\mathbf{X}_2, \mathbf{u}_2, t)$

Looking for the mean behaviour

→ ***eddy diffusivity*** **D(R,t)= d<R²>/dt**

$$\boxed{D_{Ric}(R,t) \equiv \frac{1}{2}\frac{d}{dt}\langle \mathbf{R}^2 \rangle \equiv \langle \delta_R\mathbf{u}(\mathbf{R},t)\cdot\mathbf{R}(t)\rangle \equiv \int_0^t \langle \delta_R\mathbf{u}(\mathbf{R}(t),t)\delta_R\mathbf{u}(\mathbf{R}(s),s)\rangle ds}$$

Richardson theory in the light of Kolmogorov-Obukhov theory



$$D_{Ric}(R,t) \equiv \frac{1}{2}\frac{d}{dt}\langle \mathbf{R}^2 \rangle \equiv \langle \delta_R \mathbf{u}(\mathbf{R},t) \cdot \mathbf{R}(t) \rangle \equiv \int_0^t \langle \delta_R \mathbf{u}(\mathbf{R}(t),t) \delta_R \mathbf{u}(\mathbf{R}(s),s) \rangle ds$$

local closure

multi-scale, multi-time!!

RICHARDSON 4/3 LAW

$$\boxed{D_{Ric}(R,t) \simeq \tau_R \langle \delta_R \mathbf{u}^2(\mathbf{R},t) \rangle \simeq k_0\, \epsilon^{1/3} R^{4/3}}$$

Having used Kolmogorov 1941 scaling for 3D turbulence

$$\langle \delta_r \mathbf{u}(\mathbf{r}_{||})^2 \rangle \propto \epsilon^{2/3} r^{2/3}; \qquad \tau_r \propto (r^2/\epsilon)^{1/3}$$

Richardson's theory for pair dispersion (1926)

Diffusive approach: Fokker-Planck eq.

$$\partial_t P(R,t) = \frac{1}{R^2}\frac{\partial}{\partial_R}\left[D_{Ric}(R,t)\ R^2\ \frac{\partial P(R,t)}{\partial_R}\right]$$

$D_{Ric}(R,t) \simeq R^{4/3}$ **scale-dependent** eddy diffusivity

Asymptotic Prob. Density Funtion

$$P(R,t) \simeq \frac{R^2}{t^{9/2}}\ \exp(-CR^{2/3}/t)$$

PDF P(R,t) is non-Gaussian AND non-intermittent

Second order moment

$$< R^2(t) >= g\, t^3$$

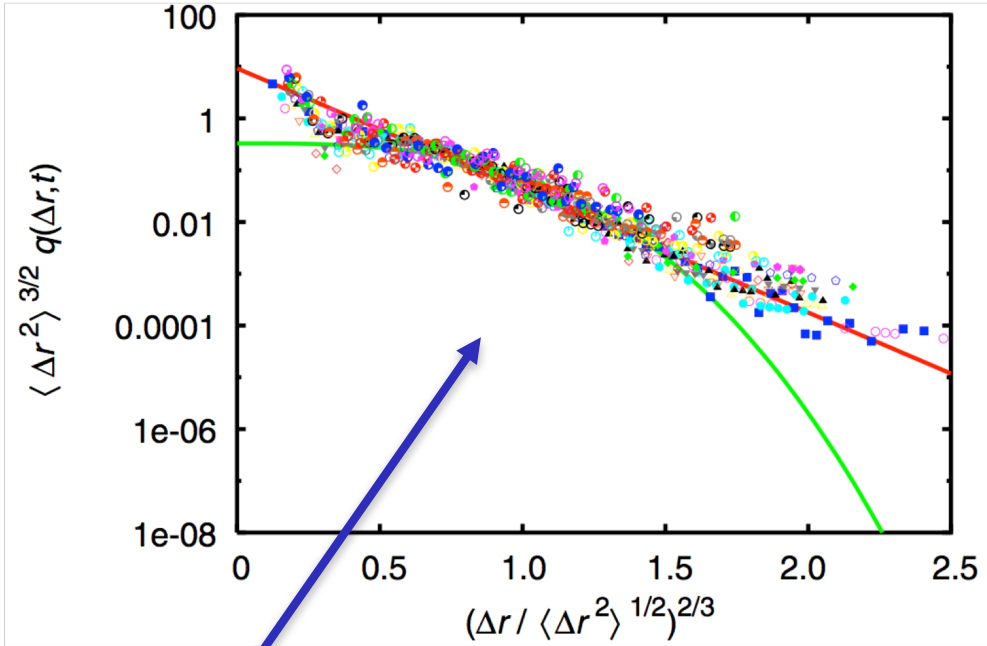
Super-Diffusive particle separation in time

Do we *observe* Richardson dispersion?

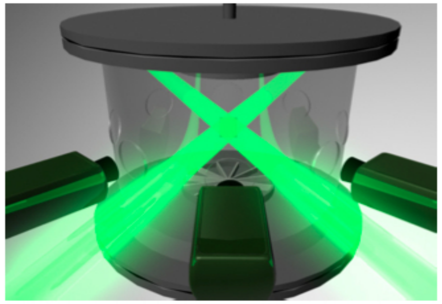
Laboratory experiments for 3D Homogeneous and Isotropic turbulence

An experimental study of turbulent relative dispersion models, Ouellette et al., New Journ. Phys. 2006

$Re_\lambda \approx 800$



$f(Hz)$	R_λ	u'_r (ms^{-1})	u'_z (ms^{-1})	ϵ (m^2s^{-3})	$\eta(\mu m)$	L/η	τ_η (ms)	T_L/τ_η	FPS	δt (ms)	$\tau_\eta/\delta t$
0.30	200	0.039	0.026	7.09×10^{-4}	192	365	36.8	51	1000	1.00	37
0.43	240	0.056	0.038	2.03×10^{-3}	146	479	21.3	61	1600	0.625	34
0.62	290	0.083	0.054	6.26×10^{-3}	111	630	12.3	74	3000	0.333	37
0.90	350	0.121	0.080	2.01×10^{-2}	84	830	7.11	88	5000	0.200	36
1.29	415	0.181	0.116	6.17×10^{-2}	64	1090	4.12	106	9000	0.111	37
1.86	500	0.262	0.169	0.196	49	1433	2.39	127	27000	0.037	65
3.50	690	0.487	0.315	1.24	30	2337	0.897	176	27000	0.037	24
5.00	815	0.669	0.440	3.39	23	3087	0.544	208	27000	0.037	15



Richardson PDF is observed but in a narrow range

POINT SOURCE EMISSION

DIRECT NUMERICAL SIMULATIONS of
3D HOMOGENEOUS AND ISOTROPIC FLOW

$$\partial_t \vec{\mathbf{u}} + (\vec{\mathbf{u}} \cdot \vec{\nabla})\,\vec{\mathbf{u}} = -1/\rho_0 \vec{\nabla} p + \nu \nabla^2 \vec{\mathbf{u}} + \vec{F}$$

Cubic periodic box 1024^3 resolution, $Re_\lambda \sim 300$

Flow is seeded with **$4*10^{11}$ pairs** for each particle family from different point sources

tracer

$$\dot{\mathbf{X}}(t) = \mathbf{u}(\mathbf{X}(t), t)$$

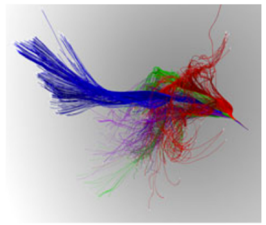
Heavy particle

$$\ddot{\mathbf{X}}_p(t) = \frac{\mathbf{u}(\mathbf{X}_p(t), t) - \dot{\mathbf{X}}_\mathbf{p}(t)}{\tau_p}$$

$St = 0.1; 0.6; 5$

Focus on Fluids journals.cambridge.org/focus

Extreme probing of particle motions in turbulence



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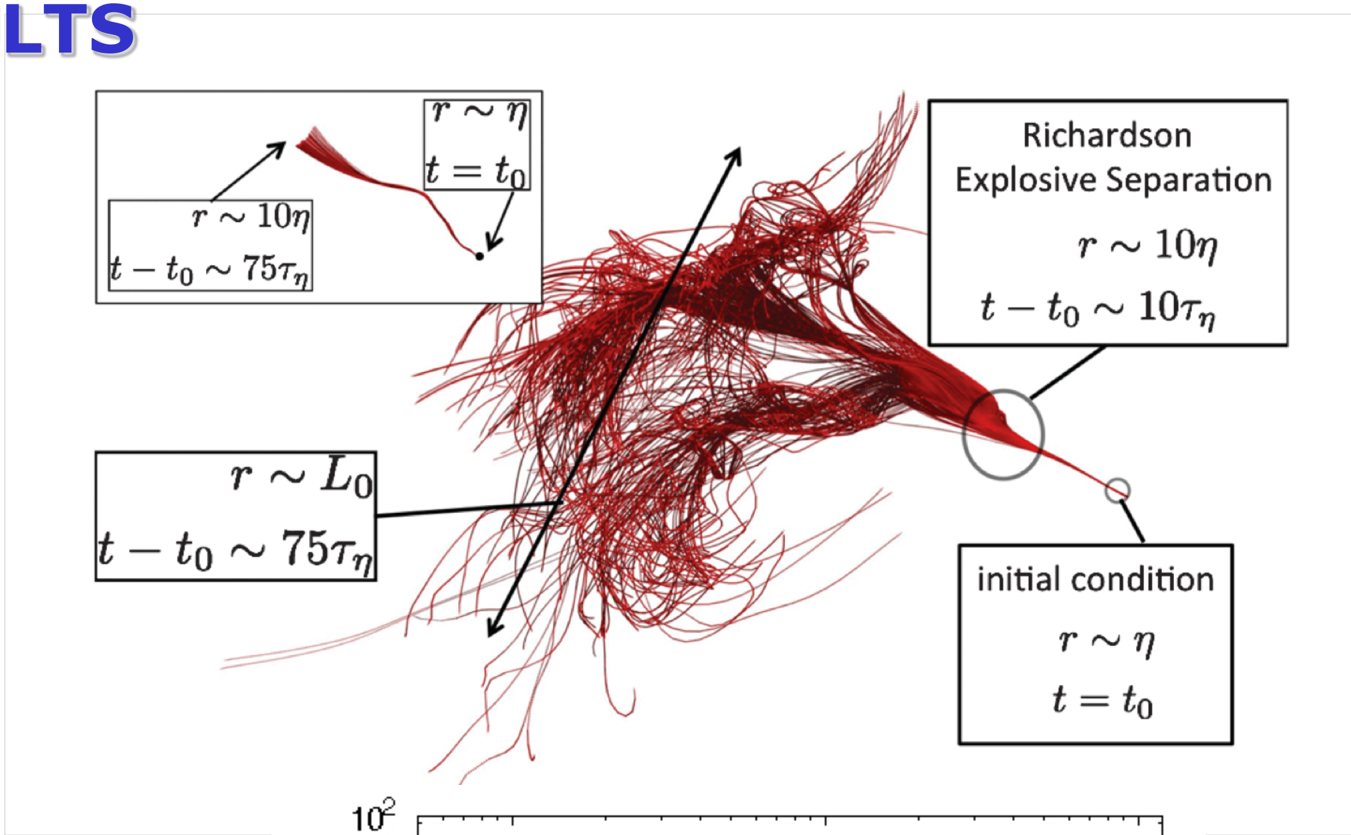
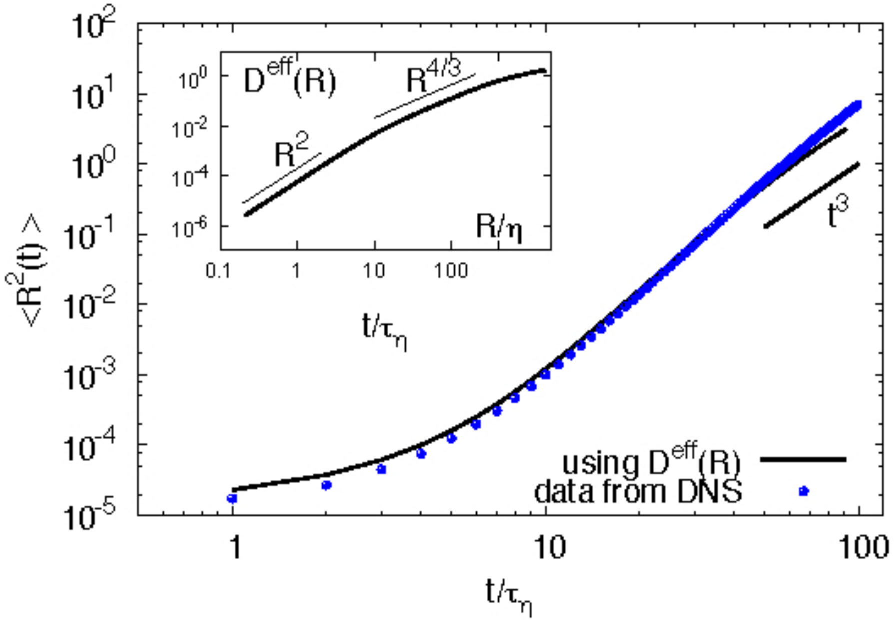
Extreme behaviour of fluid and material motions needs to be understood for engineering processes and the behaviour of clouds or plumes of pollution. Applications in the natural environment require scaling of turbulence behaviour and models beyond current computational or laboratory understanding. New computational studies of Biferale *et al.* (*J. Fluid Mech.*, 2014, vol. 757, pp. 550–572) are probing new regimes of scaling of extreme random events in nature produced by turbulent fluctuations trending towards applications in environmental prediction.

Key words: intermittency, turbulence simulation, turbulent mixing

RESULTS

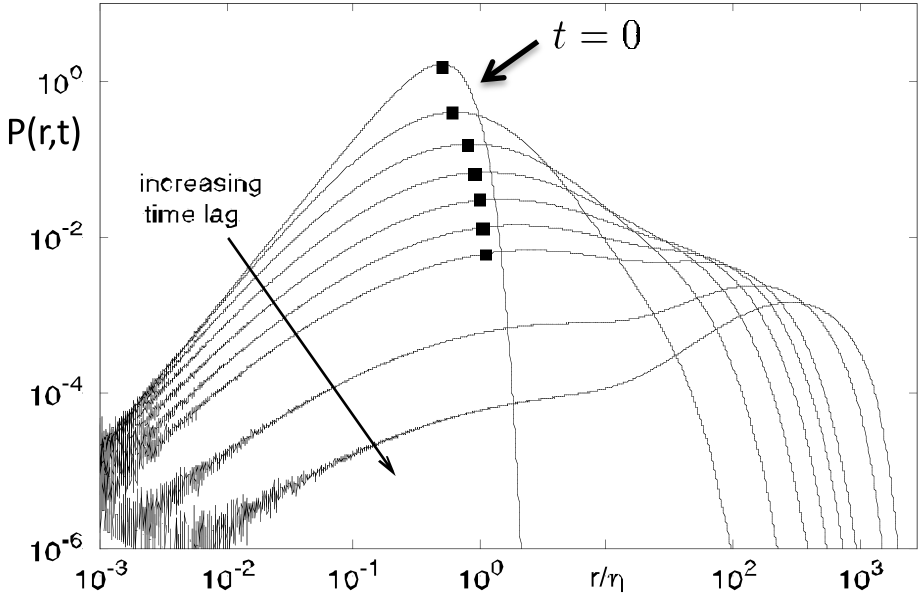


$$\langle R^2(t) \rangle = g\,\epsilon\,t^3$$



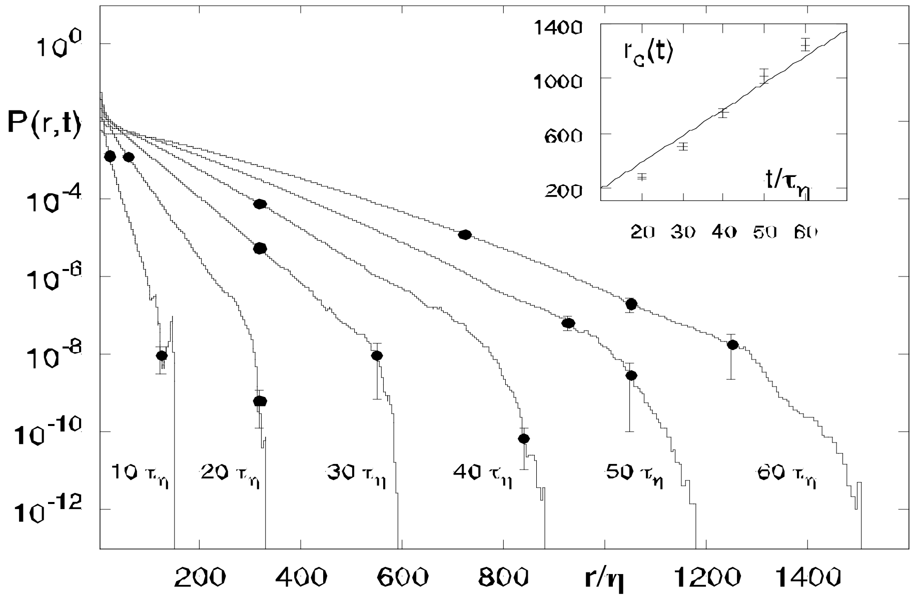
Slowest and fastest separation events

Slowly separating pairs



- Presence of a peak at sub-diffusive separations at every time.
- This behavior is due to emissions that do not separate efficiently in the flow (***very small strain at initial times***).

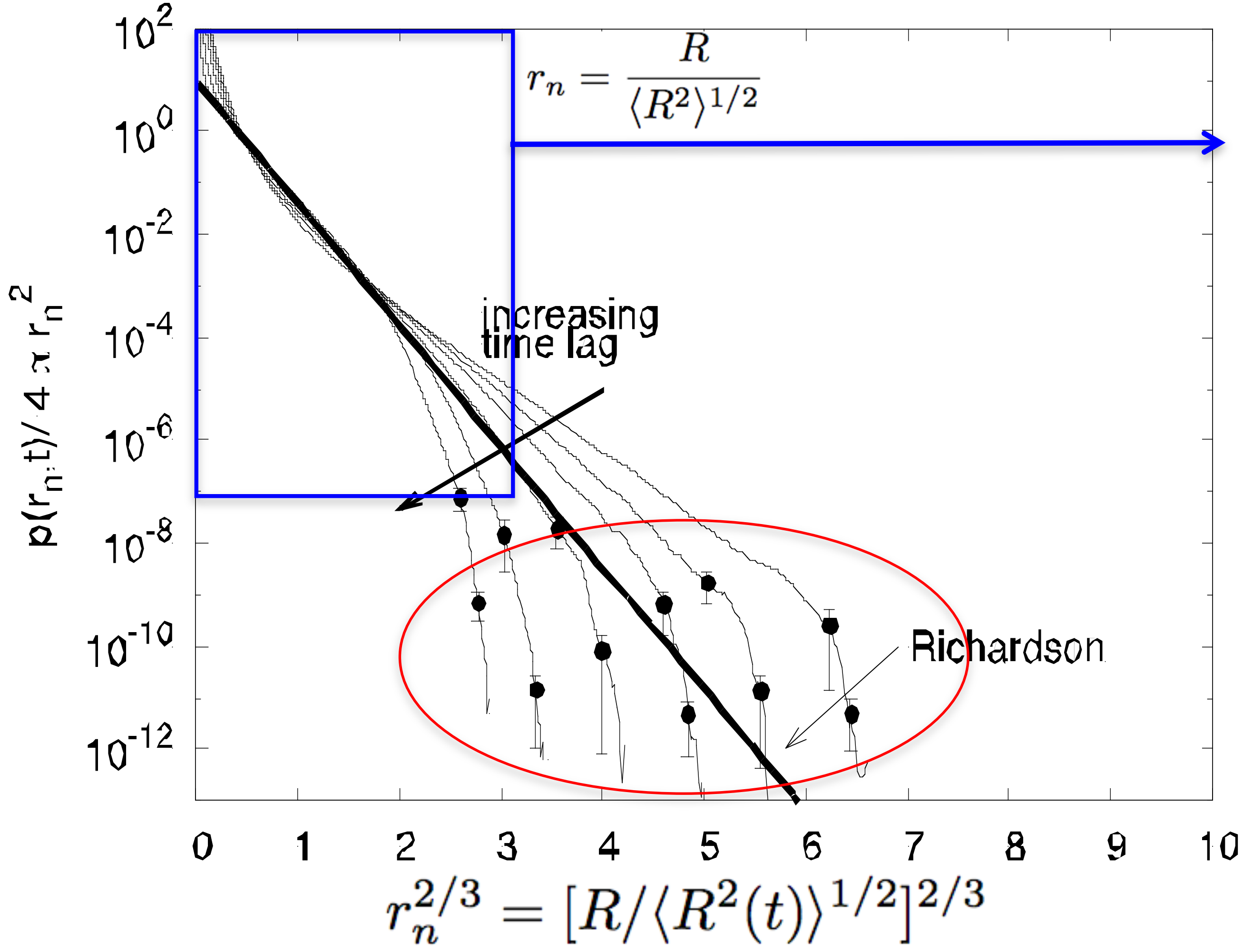
Fast separating pairs

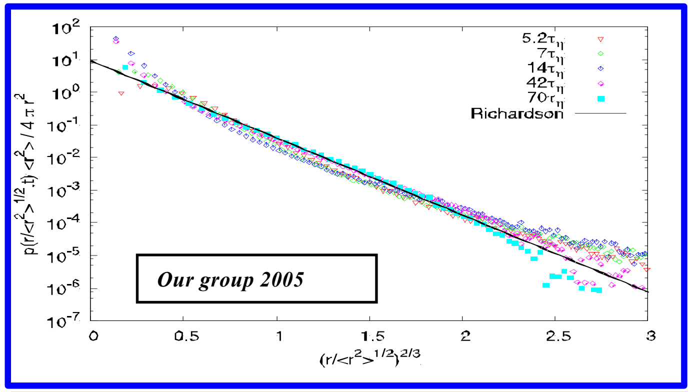


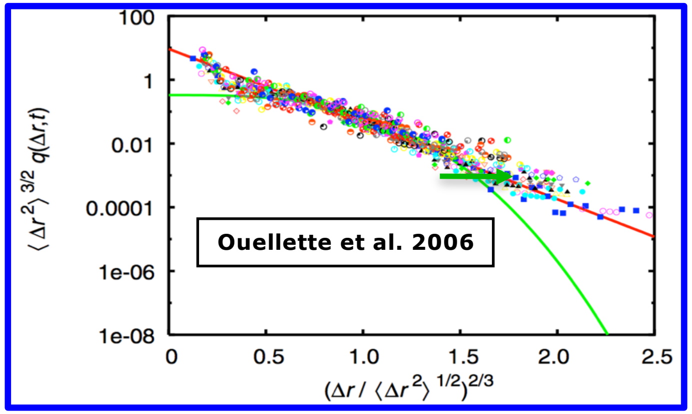
- Exponential-like tails with a sharp drop at a cut-off separation $r_c(t)$.
- This cut-off scale is the signature of ***tracer pairs experiencing a persistent high relative velocity***, which is limited by U_{rms}

PAIR SEPARATION DISTRIBUTION P(R,t)

$$\mathcal{P}\left(\frac{R}{\langle R^2\rangle^{1/2}}\right) \simeq \frac{R^2}{\langle R^2\rangle} \exp\left[-A\left(\frac{R}{\langle R^2\rangle^{1/2}}\right)^{2/3}\right]$$

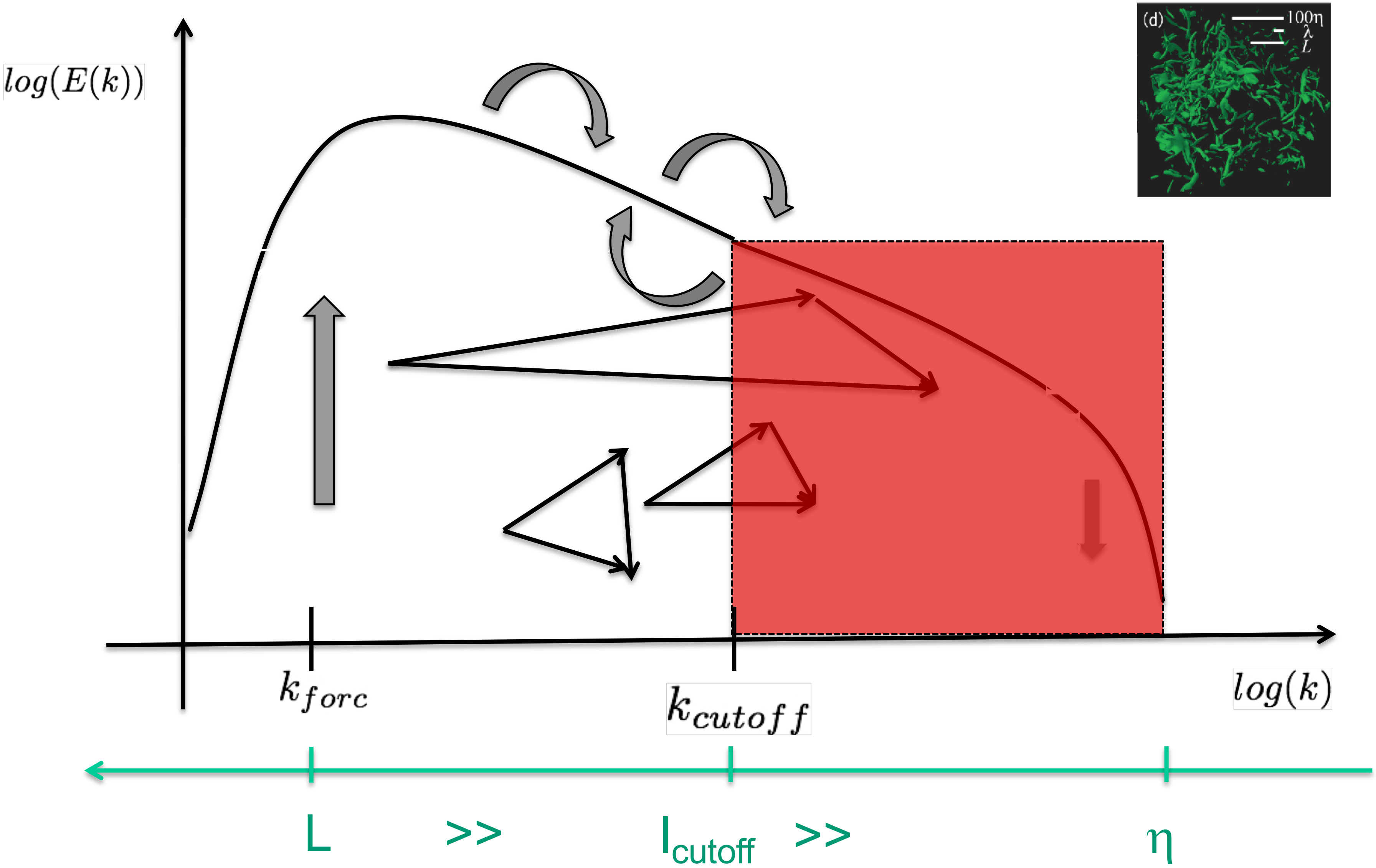




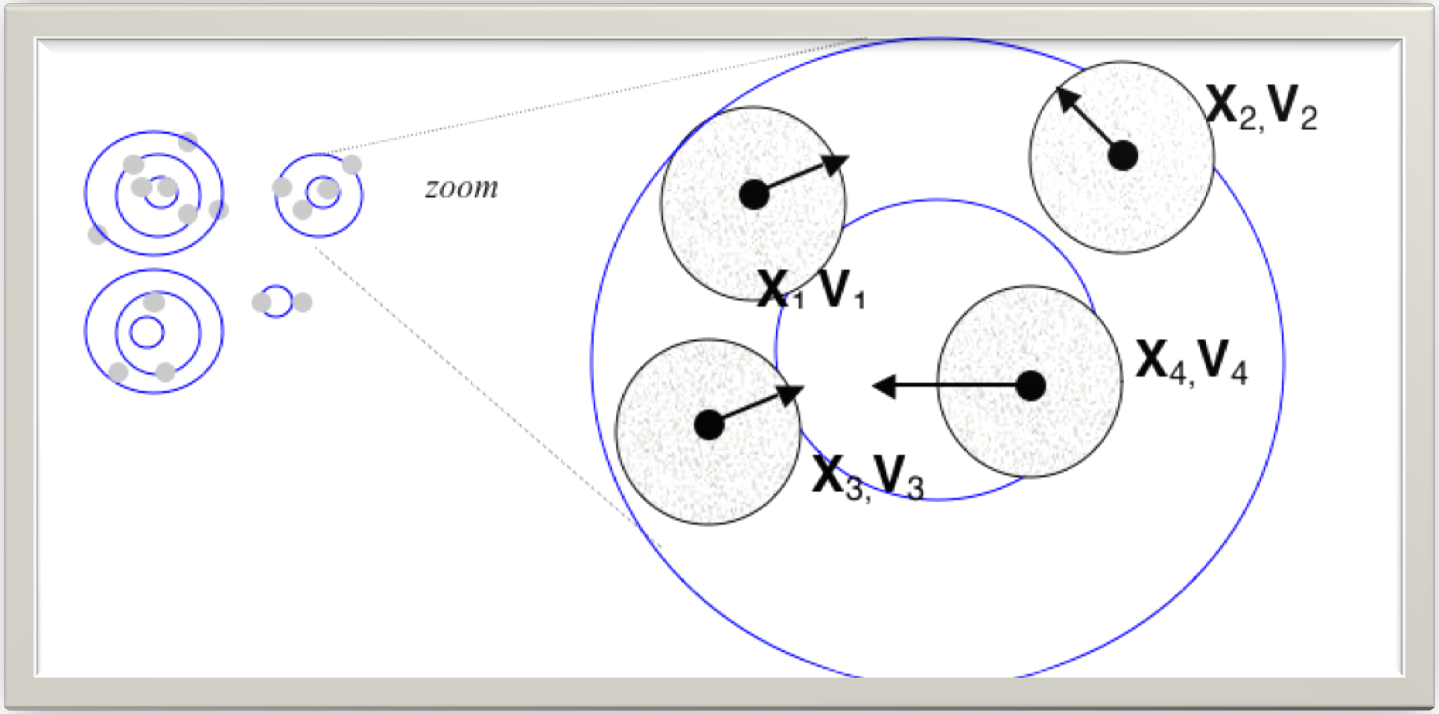


EXTREME FLUCTATIONS: are present at all times, detectable with high statistics only (***not high Re!!***)

SWITCH TO REAL WORLD!



NEW NEW: N-particle Lagrangian Sub-Grid Scale Model

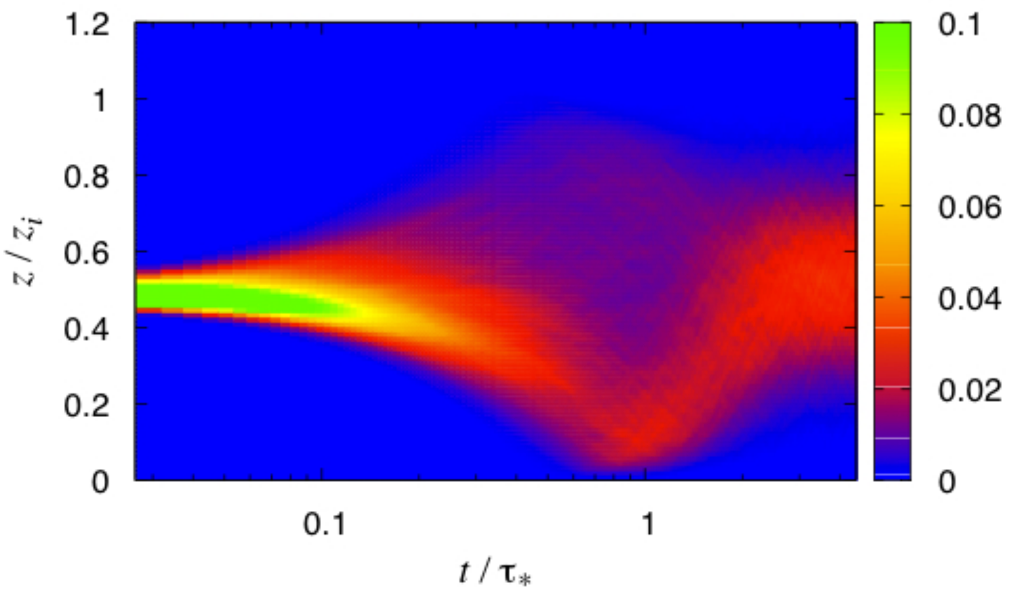


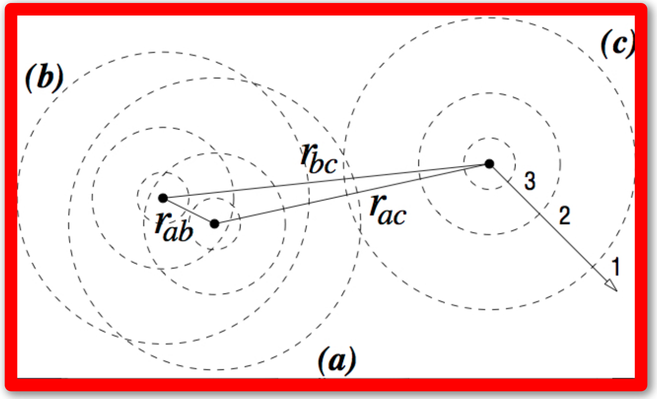
The idea is that all particles within a given **spatial scale** *l* <= *L* should be correlated with proper **time scale and energy content**.

Particles should *feel* a multiple scale velocity field, *with correct space & time properties, i.e. from Kolmogorov type of spectrum*

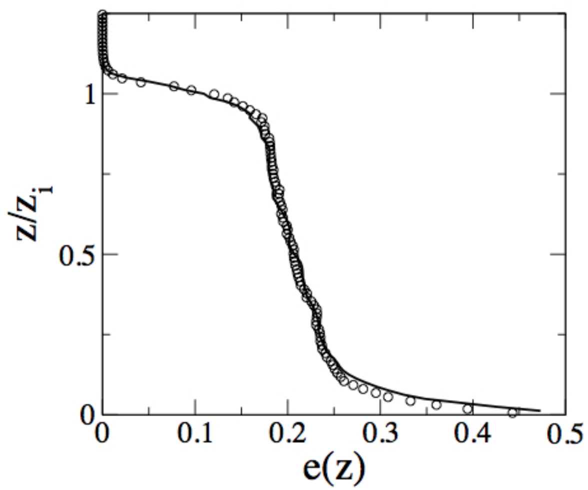
Large-eddy simulation with Lagrangian Dispersion

Emission from line source in a Convective Planetary Boundary Layer (PBL)





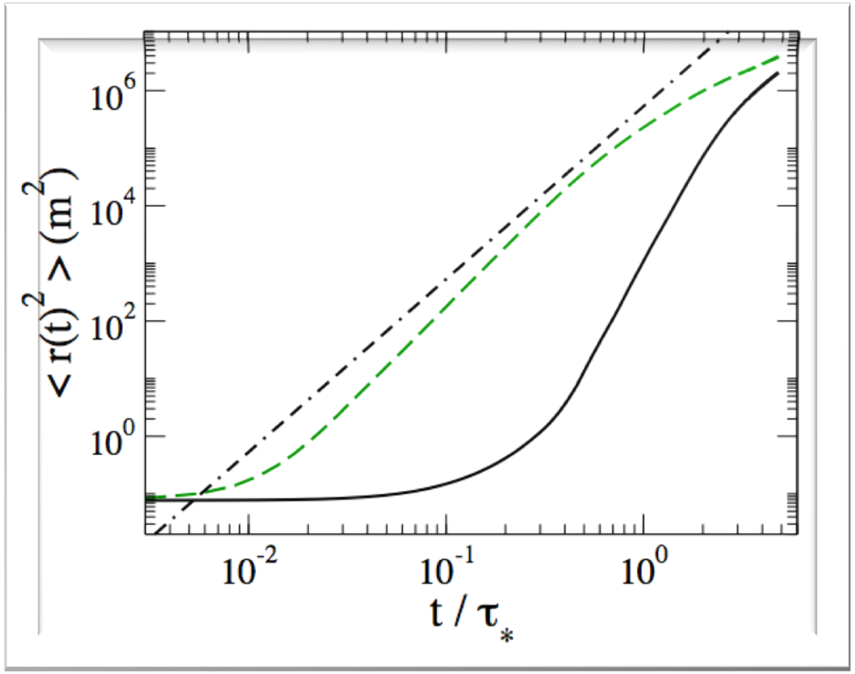
The energy dissipation rate from the SGS particle model(o) and the SGS Eulerian (line)



- 5km x 5km x 2 km domain
- 128 x 128 x 128 grid points → 39m x39m x16 m
- Dynamic SGS models for Eulerian fields
- **NEW Lagrangian SGS for tracer trajectories**

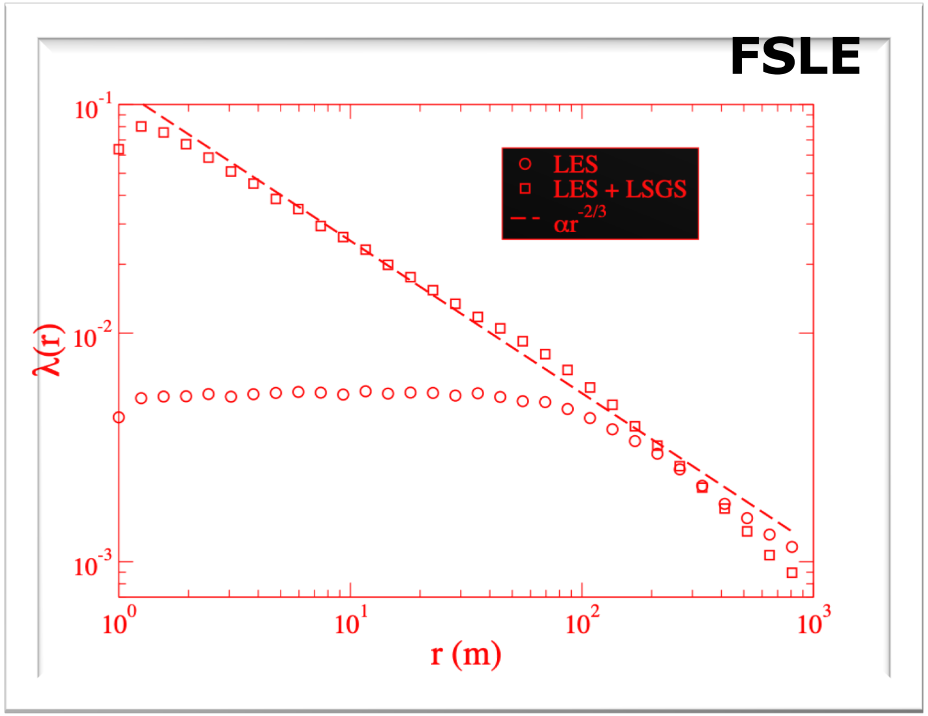
- **Npairs ~ 16000**
- **Ntetrads ~13000**
- Particle have N=28 sgs modes, from l=1m to L=78m

Results for pair separation 2nd moment



$$\langle R^2 \rangle \propto t^3$$

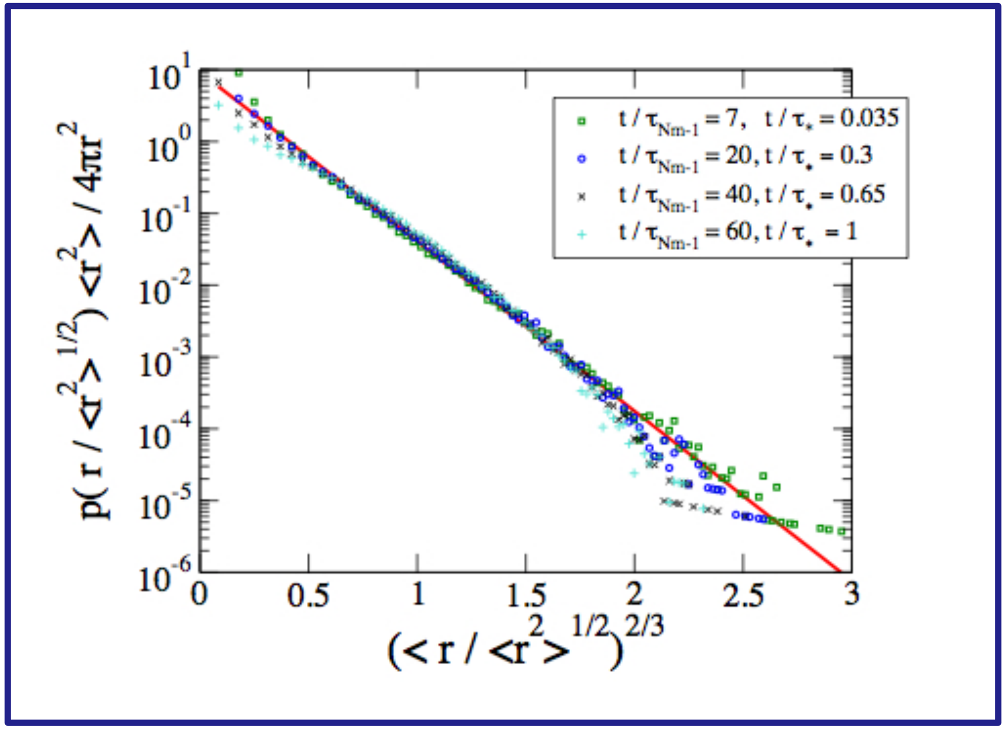
$$\lambda(R) = \frac{1}{\langle T(R) \rangle} \simeq R^{-2/3}$$



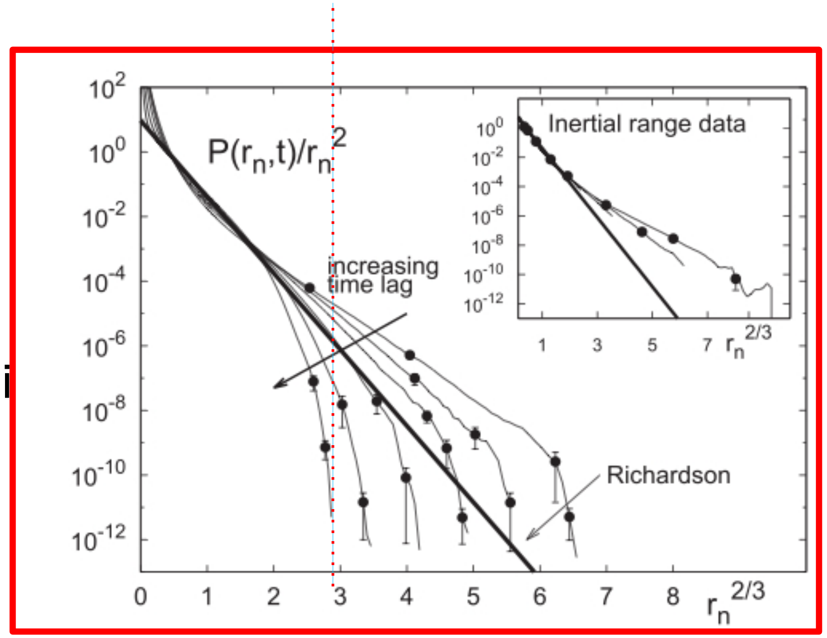
**Without the Lagrangian SGS model,
the time T(R) it takes
for a pair to separate would be
independent of R**

PAIR SEPARATION DISTRIBUTION P(R,t)

LES of Convective PBL



Richardson PDF is observed with good accuracy i
agreement with current experiments.



DNS of Homogeneous Isotropic flow

Conclusions

->**Turbulent relative dispersion is challenging**, theoretically and in applications of transport and mixing

-> **Richardson 4/3 law is the theory of reference, but not fully satisfying** difficulties arise from :

- flow intermittency
- finite Reynolds effects
- temporal correlations along Lagrangian trajectories

-> **Lagrangian Particle Models**, as the one proposed, have to correctly reproduce N-particle spatial and temporal correlations

->Challenges from real flows with **anisotropies, stratification**,…need further work

L. Biferale, A.S. Lanotte, R. Scatamacchia, F. Toschi, Journ. Fluid Mech. 757, 2014
Intermittency in the relative dispersion of tracers and heavy particles in turbulent flows

I.M. Mazzitelli ,F. Toschi, A.S. Lanotte, Phys. Fluids 26 (9) 2014
An accurate and efficient Lagrangian subgrid model

I.M. Mazzitelli, F. Fornarelli, A. S. Lanotte, P. Oresta, Phys. Fluids 26 (9) 2014
Pair and multi-particle dispersion in numerical simulations of convective boundary layer turbulence